

## **Risk Assessment by a New FMEA Model based on an Extended AHP Method under a Fuzzy Environment**

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### **Abstract**

Risk assessment has an essential role in managing different risks and their effects. A failure mode and effects analysis (FMEA), as one of the most famous risk assessment tools, has frequently been used in a wide range of industries and organizations. In this study, a new fuzzy analytic hierarchy process (AHP)-based FMEA model is introduced for evaluating the risks of various failure modes more precisely. In this model, fuzzy weighted aggregated risk priority numbers (FWARNPs) are taken into consideration instead of risk priority numbers (RPNs) for the failure modes. Moreover, considering that an economic criterion is added to the three main risk factors, the FWARNPs are calculated by utilizing four risk factors of occurrence (O), severity (S), detection (D), and cost (C). The new criterion (C) denotes the required cost for eliminating the effects of failure occurred. Also, the weights of these four risk factors are computed by an extended fuzzy AHP method. For enhancing the efficiency of the proposed model, a novel fuzzy numbers ranking method is also applied in both suggested fuzzy FMEA and AHP methods. This new ranking method is based on creating different horizontal  $\alpha$ -cuts in fuzzy numbers. Finally, to indicate the practicability and effectiveness of the proposed model, Kerman Ghete Gostar Casting Plant is considered as a case study in which the risks of toxic gas release are assessed by the suggested fuzzy FMEA model. The obtained results show that the proposed model is a practicable and advantageous risk assessment method in the real world.

**Keywords:** Risk assessment, Toxic gases release, Fuzzy FMEA, Fuzzy AHP, Fuzzy numbers ranking

### **Introduction**

Risk assessment is a comprehensive and systematic process for identifying all potential risks that may negatively affect people, property, and the environment, evaluating the detected risks, and controlling them and their effects. Considering that risk may cause human injury, damage to assets, damage to the environment, or a combination of these, assessing the risks is an important step for industries and organizations to eliminate or reduce the risks and the impacts resulting from them. There is a wide range of risk assessment methods in the literature and

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many scholars have employed these different risk assessment techniques for various case studies (Akbarpour et al., 2020; Borhani and Noorpoor, 2017; Omidvar et al., 2017; Saeidi Keshavarz et al., 2020). Therefore, selecting the appropriate method for each condition is very important. One of the most prominent risk assessment tools, which has repeatedly been utilized in a variety of applications is failure mode and effects analysis (FMEA).

The FMEA is a proactive methodology aimed at identifying and prioritizing, as much as possible, the probable risks in the area where risk assessment is performed and also their causes and related effects. Although the FMEA technique was first used in the US military and its achieved results were published in 1949, the first official application of this analysis, called the FMEA method, was in the United States aerospace industry. Afterward, it was extensively applied in the automotive and manufacturing industries respectively in the 1970s. In recent years, the FMEA methodology is employed in a variety of industries, e.g., trucking, healthcare, steel, electronic, food, and mechanical industries (Kumar Dadsena et al., 2019; Liu et al., 2019; Selim et al., 2016).

In the classical FMEA method, a criterion named risk priority number (RPN) is utilized to calculate the risks of different failure modes. The RPN is computed by the arithmetic product of three risk factors, i.e., occurrence (O), severity (S), and detection (D). A ten-point scale is applied for evaluating the three mentioned risk factors in the calculations related to the conventional RPN. The greater value of the RPN means the higher risk priority level of the corresponding failure mode. Despite the widespread application of the traditional FMEA, it has major drawbacks which some of the most important ones are as follows (Gul et al., 2020; Huang et al., 2020; Park et al., 2018):

- The weights of the risk factors of occurrence, severity, and detection are not taken into consideration.
- Different ratings of the three risk factors O, S, and D may exactly yield the same RPN value even though their real risk levels may be different. For instance, given that the values of O, S, and D for two different failure modes are 5, 4, 3 and 2, 3, 10, respectively, both of the failure modes have the same RPN value, i.e., 60.
- Only the three risk factors of occurrence, severity, and detection are considered, whereas other important parameters, such as economic parameters, are ignored.
- It is difficult for experts to estimate the values of the three risk factors O, S, and D precisely due to the use of crisp numbers in the ten-point scale for assessing the risk factors.

In recent years, many approaches have been introduced by different scholars to conquer the above-mentioned shortcomings of the conventional FMEA method (Bhattacharjee et al., 2020; Boral et al., 2020; Geramian et al., 2020; Qin et al., 2020). One of these approaches is to utilize fuzzy set theory in the FMEA technique. The fuzzy set theory was firstly presented by Zadeh (1965). For using ambiguous and inexact data and information, the fuzzy set theory can be employed in the FMEA methodology, in which the risk factors are assessed by decision-makers (DMs) utilizing fuzzy linguistic terms.

In previous studies, various researchers have applied the fuzzy approach in the FMEA method. Baykasoğlu and Gölcük (2020) introduced a new comprehensive fuzzy FMEA model to assess the risks of an enterprise resource planning (ERP) implementation in a software producer company. In their proposed hybrid model, three fuzzy approaches of fuzzy preference programming (FPP), fuzzy cognitive maps (FCMs), and fuzzy graph-theoretical matrix approach (FGTMA) were combined. In the work of Rezaee et al. (2020), a novel integrated method based on linguistic FMEA, fuzzy inference system (FIS), and slack-based data envelopment analysis (SBDEA) model was presented for evaluating health, safety, and environment (HSE) risks in an active company in the chemical industry. They employed the fuzzy inference system to reach a general agreement on different values of the risk factors determined by FMEA team members.

Li and Chen (2019) suggested a new evidential FMEA methodology in which fuzzy belief structure and grey relational projection method (GRPM) were integrated for risk assessment in a steel factory. In their method, the fuzzy belief structure was used to demonstrate the opinions of experts more rationally and flexibly. Also, they proposed a novel approach for transforming various fuzzy evaluations of experts into basic probability assignments (BAPs). In the study of Wan et al. (2019), a new hybrid fuzzy FMEA model based on the Bayesian network (BN) technique was developed to evaluate maritime supply chain risks in a real case study of a container shipping company. In the suggested model, they combined the Bayesian network method with fuzzy rule-based risk inference for incorporating subjective judgments into the process of risk assessment under uncertainty.

Another efficient approach for removing some of the mentioned deficiencies of the traditional FMEA method is to combine the FMEA with multi-criteria decision-making (MCDM) techniques. The MCDM is known as a sub-discipline of operations research that is employed for ranking and prioritizing a set of alternatives based on several criteria (Padash, 2017). One of the most extensively utilized MCDM methods is the analytic hierarchy process (AHP) technique (Amini et al., 2020). The AHP technique was introduced by Saaty (1980) for the first time. It provides a logical framework for organizing and evaluating complex decisions by applying mathematics and psychology (Padash and Ghatari, 2020). The AHP method consists of three main components, namely the ultimate objective that should be achieved or the problem that should be solved, a set of the possible solutions, named alternatives, and the criteria and sub-criteria that the alternatives should be judged on them (Padash and Atae, 2019). These judgments are implemented by the decision-makers through pairwise comparisons (Vahidi et al., 2014). One of the advantages of the AHP technique is its flexibility in these comparisons (Nejad et al., 2013).

The fuzzy set theory can also be applied in the MCDM methods to use ambiguous and inexact data and information. In other words, fuzzy MCDM techniques are utilized for adding uncertainty and ambiguity to the systems and enhancing the calculations' flexibility (Seiti et al., 2018). Considering that the conventional AHP method cannot completely reflect the ambiguous feeling and recognition of experts, a fuzzy AHP (FAHP) approach was introduced to resolve the weaknesses of the conventional AHP (Padash et al., 2021; Padash et al., 2020). For the first time, the FAHP method was presented by Van Laarhoven and Pedrycz (1983).

In the literature, many scholars have combined the FMEA method with the AHP technique. Sadeghi et al. (2021) proposed a new hybrid approach, in which the FMEA and AHP methods were integrated to assess environmental risks in Kahrizak landfill of Tehran. In their approach, the AHP and FMEA methods were utilized for determining the severity of the detected risks and evaluating these risks, respectively. In the work of Yucesan and Gul (2021), a new FMEA model based on neutrosophic AHP (NAHP) was presented to resolve some of the deficiencies of the traditional FMEA method. In this model, they first merged the AHP method with neutrosophic sets for calculating the weights of the risk factors in the FMEA method. Then, they prioritized the identified failure modes with respect to these weighted risk factors. Karatop et al. (2021) introduced a novel hybrid model based on the fuzzy FMEA, the analytic hierarchy process, and the evaluation based on distance from average solution (EDAS) methods for determining the optimum investments in the renewable energy sector of Turkey. In their suggested hybrid approach, the FAHP and EDAS methods were employed to calculate the importance weights of renewable energy alternatives.

Wang et al. (2021) suggested a novel approach on the basis of the FMEA method and a cloud model (CM) for risk assessment of a coal-to-methanol plant in China. In this approach, the interval AHP (IAHP) method and the CM were used to compute the cloud weights of the three risk factors of severity, occurrence, and detection. Afterward, the cloud RPNs (CRPNs) were calculated instead of the RPNs to enhance the effectiveness of the conventional RPN. In the

research of Boral et al. (2020), a new integrated fuzzy FMEA methodology based on the fuzzy analytic hierarchy process and an improved fuzzy multi-attribute ideal real comparative analysis (FMAIRCA) was proposed to enhance the effectiveness of the risk evaluation process. They used the FAHP and modified FMAIRCA methods for computing the relative importance of the risk factors and prioritizing the failure modes, respectively. Hassan et al. (2019) presented a hybrid risk assessment approach in which the FMEA method was combined with the FAHP technique to detect and mitigate potential process failures in the warehouse of the cement industry in Indonesia. In their suggested approach, the FAHP method was applied for decreasing subjectivity in the weighting process.

As it can be observed from the above-mentioned papers, numerous researchers have tried to eliminate the weaknesses of the classical FMEA method. However, many other works can still be performed in this case. For this reason, this research proposes a new fuzzy FMEA model based on an extended fuzzy AHP method to resolve the mentioned shortcomings of the traditional FMEA and assess the risks of different failure modes more accurately. In the suggested hybrid model, a new risk assessment criterion, namely fuzzy weighted aggregated risk priority number (FWARN) is utilized instead of the RPN for evaluating the risk of each failure mode. Furthermore, considering that in the conventional FMEA, only the safety parameters of occurrence, severity, and detection are taken into consideration and other important parameters, for instance, economic parameters, are ignored, in the proposed model, a new economic parameter named cost (C) is added to the three previous risk parameters O, S, and D. This parameter denotes the required cost for eliminating the effects of failure occurred. Accordingly, the FWARNs are computed by using the four risk parameters O, S, D, and C. Also, the suggested extended fuzzy AHP method is employed for calculating the weights of these four risk parameters. Additionally, a new fuzzy numbers ranking method is used in both the proposed fuzzy FMEA and AHP methods to improve the efficacy of the suggested model.

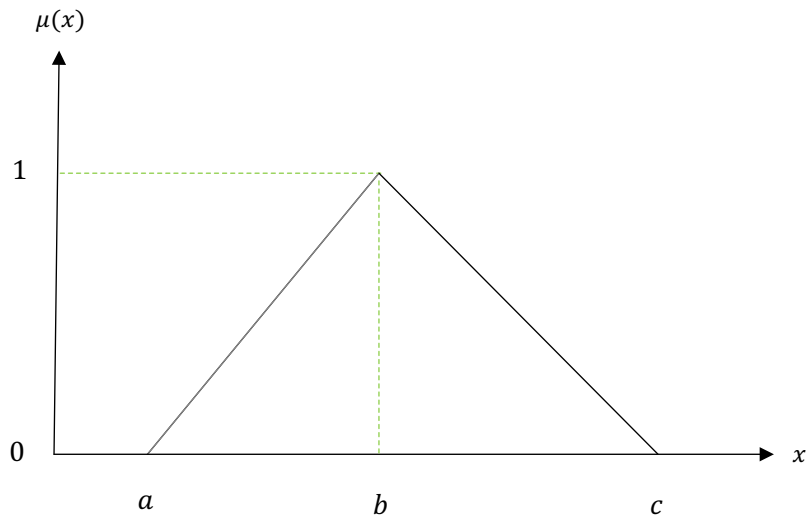
This paper consists of four sections including the mentioned section of the introduction. In the second section of the paper, the fuzzy set theory, the proposed ranking approach of fuzzy numbers, the suggested extended fuzzy AHP method, and the proposed FAHP-based FMEA model are presented, respectively. In the third section, the feasibility of the suggested model is indicated through a real case study of a casting plant. In the final section, this study is concluded.

## Material and methods

### *Fuzzy set theory*

Zadeh (1965) proposed fuzzy set theory to deal with the ambiguity in decision-making processes. In the fuzzy set theory, a membership function characterizes a fuzzy set. The membership function of the fuzzy set can be demonstrated in various ways of which the most commonly used ones are triangular and trapezoidal. In this research, triangular fuzzy numbers are applied. As can be observed from figure 1, a triangular fuzzy number can be stated as  $\tilde{A} = (a, b, c)$ . The membership function of triangular fuzzy numbers can be showed by the following equation:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}; & a \leq x \leq b \\ \frac{c-x}{c-b}; & b \leq x \leq c \\ 0; & \text{Otherwise} \end{cases} \quad (1)$$



**Figure 1.** Triangular fuzzy number.

The basic algebraic operations for two positive triangular fuzzy numbers  $\tilde{A} = (a, b, c)$  and  $\tilde{B} = (d, e, f)$  and a positive real number  $r$  can be expressed as the following equations (Fattahi and Khalilzadeh, 2018):

$$\tilde{A} \oplus \tilde{B} = [a + d, b + e, c + f] \quad (2)$$

$$\tilde{A} \ominus \tilde{B} = [a - f, b - e, c - d] \quad (3)$$

$$\tilde{A} \otimes \tilde{B} \cong [ad, be, cf] \quad (4)$$

$$\tilde{A} \otimes r = [ar, br, cr] \quad (5)$$

$$\tilde{A}^{-1} \cong [1/c, 1/b, 1/a] \quad (6)$$

#### *Proposed novel fuzzy numbers ranking method*

Ranking fuzzy numbers helps the decision-makers to find the best alternative about ambiguous information, thus, it plays an essential role in decision-making processes. Two fuzzy numbers should be compared with their equivalent crisp numbers. A suitable and effective ranking method of fuzzy numbers must keep the characteristics of fuzzy numbers when being converted into crisp numbers. In this research, a novel method is suggested for ranking triangular fuzzy numbers. It is a simple and accurate method for ranking triangular fuzzy numbers. The steps of this method can be stated as follows:

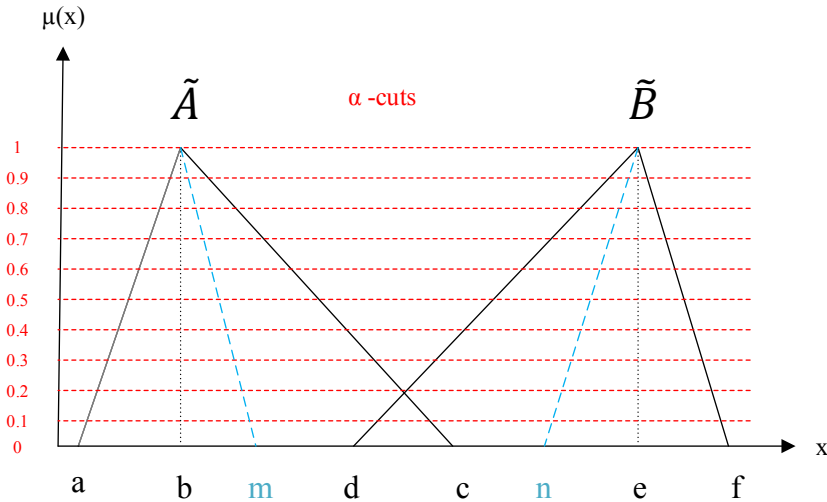
Consider the two triangular fuzzy numbers  $\tilde{A} = (a, b, c)$  and  $\tilde{B} = (d, e, f)$  displayed in figure 2.

Step 1. The midpoint of the side  $ac$  is calculated by utilizing the following equation:

$$m = (a + c)/2 \quad (7)$$

Step 2. The line  $bm$  is depicted between the points  $b$  and  $m$ .

Step 3. Ten horizontal  $\alpha$  – cuts with equal distances are drawn.



**Figure 2.** Two triangular fuzzy numbers  $\tilde{A} = (a, b, c)$  and  $\tilde{B} = (d, e, f)$ .

Step 4. The coordinates of the intersection of each  $\alpha$  – cut with the line  $bm$  are computed by employing the following equations:

$$ym_i = 0.1 * i, \quad i = 1, 2, \dots, 10 \quad (8)$$

$$xm_i = ((b - m) * ym_i) + m, \quad i = 1, 2, \dots, 10 \quad (9)$$

Step 5. The equivalent crisp number of the triangular fuzzy number  $\tilde{A}$  is calculated by applying the following equation:

$$R_{\tilde{A}} = \sum_{i=1}^{10} (xm_i * ym_i^{0.01}) \quad (10)$$

Step 6. Steps 1 to 5 are also performed for the triangular fuzzy number  $\tilde{B}$ , and its equivalent crisp number is computed as  $R_{\tilde{B}}$ .

Step 7. The ratio of  $\tilde{A}$  and  $\tilde{B}$  is calculated by dividing the two crisp numbers  $R_{\tilde{A}}$  and  $R_{\tilde{B}}$ :

$$R_{(\tilde{A}/\tilde{B})} = \frac{R_{\tilde{A}}}{R_{\tilde{B}}} \quad (11)$$

In the next section, to show the validity and efficiency of the suggested fuzzy numbers ranking method, one comparative example is provided.

### Comparative example

Example. In this comparative example, four commonly used fuzzy sets are considered. In other words, in this example, the triangular fuzzy numbers of these fuzzy sets are compared by the suggested fuzzy numbers ranking method and six famous methods presented by different researchers. Table 1 demonstrates these methods and the results obtained by them. The four sets are as follows:

Set 1:  $\tilde{A} = (1, 1, 3)$ ,  $\tilde{B} = (1, 1, 7)$

Set 2:  $\tilde{A} = (2, 4, 6)$ ,  $\tilde{B} = (1, 5, 6)$ ,  $\tilde{C} = (3, 5, 6)$

Set 3:  $\tilde{A} = (2, 3, 8)$ ,  $\tilde{B} = (2, 3, 10)$

Set 4:  $\tilde{A} = (2, 4, 6)$ ,  $\tilde{B} = (1, 5, 6)$

**Table 1.** Results of ranking the four mentioned sets by the suggested method and six famous methods.

| Method                        | Set 1                      | Set 2                                  | Set 3                      | Set 4                   |
|-------------------------------|----------------------------|----------------------------------------|----------------------------|-------------------------|
| Wang et al. (2009)            | $\tilde{A} \sim \tilde{B}$ | $\tilde{A} \sim \tilde{B} < \tilde{C}$ | $\tilde{A} < \tilde{B}$    | $\tilde{B} < \tilde{A}$ |
| Abbasbandy and Hajjary (2009) | $\tilde{A} < \tilde{B}$    | $\tilde{A} < \tilde{B} < \tilde{C}$    | $\tilde{A} < \tilde{B}$    | $\tilde{A} < \tilde{B}$ |
| Cheng (1998)                  | $\tilde{A} < \tilde{B}$    | $\tilde{A} < \tilde{B} < \tilde{C}$    | $\tilde{A} < \tilde{B}$    | $\tilde{A} < \tilde{B}$ |
| Chu and Tsao (2002)           | $\tilde{A} < \tilde{B}$    | $\tilde{A} < \tilde{B} < \tilde{C}$    | $\tilde{A} < \tilde{B}$    | $\tilde{A} < \tilde{B}$ |
| Deng et al. (2006)            | $\tilde{A} < \tilde{B}$    | $\tilde{A} < \tilde{B} < \tilde{C}$    | $\tilde{A} < \tilde{B}$    | $\tilde{A} < \tilde{B}$ |
| Nejad and Mashinchi (2011)    | $\tilde{A} < \tilde{B}$    | $\tilde{A} < \tilde{B} < \tilde{C}$    | $\tilde{A} \sim \tilde{B}$ | $\tilde{A} < \tilde{B}$ |
| The proposed method           | $\tilde{A} < \tilde{B}$    | $\tilde{A} < \tilde{B} < \tilde{C}$    | $\tilde{A} < \tilde{B}$    | $\tilde{A} < \tilde{B}$ |

The results demonstrate that though the proposed novel method is simpler than the previous methods, it precisely performs the comparisons of the four mentioned sets.

*The proposed extended fuzzy AHP method*

First, it should be mentioned that the major difference between this method and the conventional fuzzy AHP method is that the suggested novel fuzzy numbers ranking approach, which its accuracy was proven in the previous section, is utilized in the proposed method to improve its effectiveness and precision. The steps of the suggested method are as follows:

Step 1. The hierarchical structure of the problem is established based on the main objective of the decision-making process.

Step 2. The fuzzy pairwise comparison matrices for the criteria for the objective are constructed by the decision-makers (DMs) considering Table 2.

Step 3. The consistency of each fuzzy pairwise comparison matrix established in the previous step is checked by applying the method suggested by Gogus and Boucher (1998) for validating each of the judgment matrices. Based on this method, first, two values  $CR^m$  and  $CR^g$  which are defined as the consistency ratio of the matrix of mean values and the consistency ratio of the matrix of geometric means of lower and upper bounds, respectively, are computed for each fuzzy pairwise comparison matrix. Afterward, the matrix is accepted if both of these values ( $CR^m$  and  $CR^g$ ) achieved for it, are 0.1 or less than 0.1.

**Table 2.** Fuzzy evaluation scores (Fattahi et al., 2020).

| Linguistic variable | Crisp number | Triangular fuzzy number | Reciprocal triangular fuzzy number |
|---------------------|--------------|-------------------------|------------------------------------|
| Equal               | 1            | (1,1,1)                 | (1,1,1)                            |
| Moderate            | 3            | (1,1,1.5)               | (2/3,1,1)                          |
| Strong              | 5            | (1,1.5,2)               | (1/2,2/3,1)                        |
| Very strong         | 7            | (1.5,2,2.5)             | (2/5,1/2,2/3)                      |
| Extreme             | 9            | (2,2.5,3)               | (1/3,2/5,1/2)                      |

Step 4. The fuzzy pairwise comparison matrices constructed in Step 2 are aggregated by using the following equation:

$$\tilde{c}_{ij} = (c_{ij1}, c_{ij2}, c_{ij3}) = \left( \frac{1}{l} \sum_{k=1}^l c_{ij1}^k, \frac{1}{l} \sum_{k=1}^l c_{ij2}^k, \frac{1}{l} \sum_{k=1}^l c_{ij3}^k \right), \quad i, j = 1, 2, \dots, n \tag{12}$$

where the element  $\tilde{c}_{ij}^k$  indicates the fuzzy relative importance of the criterion  $C_i$  ( $i = 1, 2, \dots, n$ ) over the criterion  $C_j$  ( $j = 1, 2, \dots, n$ ) determined by the decision-maker  $DM_k$  ( $k = 1, 2, \dots, l$ ).

Step 5. The summation of the triangular fuzzy numbers of each column in the fuzzy group pairwise comparison matrix established in the previous step is computed by Eq. (2).

Step 6. The ratio of each triangular fuzzy number with the summed fuzzy value of the triangular fuzzy numbers of its column achieved from the previous step is calculated by utilizing the proposed new fuzzy numbers ranking method.

Step 7. The weight of each criterion is obtained by computing the arithmetic mean of the ratios of each row calculated in the previous step.

### *Proposed new fuzzy FMEA model*

The steps of the suggested model are as follows:

Step 1. Potential failure modes are detected.

Step 2. The fuzzy values of the four risk factors of occurrence (O), severity (S), detection (D), and cost (C) (the required cost for eliminating the effects of failure occurred) for the identified failure modes are determined by the decision-makers considering Tables 3 and 4 (Fattahi and Khalilzadeh, 2018).

Step 3. The fuzzy values of each risk factor determined by different DMs for each failure mode are aggregated similar to the fourth step of the suggested extended fuzzy AHP method.

**Table 3.** Fuzzy rating scales for occurrence and severity.

| Occurrence (O)                            | Fuzzy rating | Severity (S)           | Fuzzy rating |
|-------------------------------------------|--------------|------------------------|--------------|
| Certain probability of occurrence         | (9, 10, 10)  | Extremely dangerous    | (9, 10, 10)  |
| Failure is almost inevitable              | (8, 9, 10)   | Very dangerous         | (8, 9, 10)   |
| Very high probability of occurrence       | (7, 8, 9)    | Dangerous              | (7, 8, 9)    |
|                                           | (6, 7, 8)    |                        | (6, 7, 8)    |
| Moderately high probability of occurrence | (5, 6, 7)    | Moderate danger        | (5, 6, 7)    |
|                                           | (4, 5, 6)    |                        | (4, 5, 6)    |
| Moderate probability of occurrence        | (3, 4, 5)    | Low to moderate danger | (3, 4, 5)    |
|                                           | (2, 3, 4)    |                        | (2, 3, 4)    |
| Low probability of occurrence             | (1, 2, 3)    | Slight danger          | (1, 2, 3)    |
| Remote probability of occurrence          | (1, 1, 2)    | No danger              | (1, 1, 2)    |

**Table 4.** Fuzzy rating scales for detection and cost.

| Detection (D)                              | Fuzzy rating | Cost (C)             | Fuzzy rating |
|--------------------------------------------|--------------|----------------------|--------------|
| No chance of detection                     | (9, 10, 10)  | Extremely costly     | (9, 10, 10)  |
| Very remote/unreliable chance of detection | (8, 9, 10)   | Very costly          | (8, 9, 10)   |
|                                            | (7, 8, 9)    |                      | (7, 8, 9)    |
| Remote chance of detection                 | (6, 7, 8)    | Costly               | (6, 7, 8)    |
|                                            | (5, 6, 7)    |                      | (5, 6, 7)    |
| Moderate chance of detection               | (4, 5, 6)    | Moderate cost        | (4, 5, 6)    |
| High chance of detection                   | (3, 4, 5)    | Low to moderate cost | (3, 4, 5)    |
|                                            | (2, 3, 4)    |                      | (2, 3, 4)    |
| Very high chance of detection              | (1, 2, 3)    | Slight cost          | (1, 2, 3)    |
| Almost certain chance of detection         | (1, 1, 2)    | No cost              | (1, 1, 2)    |

Step 4. The weights of the four risk factors O, S, D, and C are computed by the proposed extended fuzzy AHP method.

Step 5. The fuzzy weighted aggregated values of the risk factors are calculated for each failure mode by multiplying the fuzzy aggregated value of each risk factor by the weight of the risk factor using Eq. (5).



Step 6. The fuzzy weighted aggregated risk priority number (FWARNP) is computed for each failure mode by multiplying the fuzzy weighted aggregated values of the four risk factors O, S, D, and C utilizing Eq. (4).

Step 7. The detected failure modes are ranked by comparing the obtained FWARNPs applying the suggested new fuzzy numbers ranking method.

## Application and results

To demonstrate the practicability and effectiveness of the suggested fuzzy FMEA model, Kerman Ghete Gostar Casting Plant was selected as a case study in the real world. This plant was constructed in Khazra Industrial Park of Kerman in 2007. The plant aims to produce different parts and equipment needed for a variety of mining and industrial companies in the country. At present, it engages in the casting of various alloys such as shatterproof alloys, alloy steel, incombustible alloys, and alloy cast iron.

In the mentioned real case study, the risks of toxic gas release considered as failure modes were evaluated. For this purpose, the steps of the proposed novel fuzzy FMEA model expressed in the previous section were respectively implemented in the plant. In other words, in the first step, different toxic gases that may be released in the plant were identified by the decision-making team consisted of three experienced experts of the plant, namely one operations manager ( $DM_1$ ), one production manager ( $DM_2$ ), and one health, safety, and environment (HSE) inspector ( $DM_3$ ). These DMs detected seven toxic gases that may be released in different processes of the plant. The names and chemical formulas of these identified toxic gases can be mentioned as follows: sulfur dioxide ( $SO_2$ ), cyanogen ( $C_2N_2$ ), chlorine ( $Cl$ ), nitrogen dioxide ( $NO_2$ ), carbonyl chloride (phosgene) ( $COCl_2$ ), hydrogen sulfide ( $H_2S$ ), and carbon monoxide ( $CO$ ).

In the next step, the fuzzy values of the four risk factors O, S, D, and C for the detected toxic gases were determined by the three decision-makers  $DM_1$ ,  $DM_2$ , and  $DM_3$  considering Tables 3 and 4. Table 5 shows these fuzzy values. Afterward, the fuzzy values of each risk factor determined by the three DMs for each toxic gas were aggregated. Then, the weights of the four risk factors O, S, D, and C were calculated by the suggested extended fuzzy AHP method. To this aim, first, the hierarchical structure of the problem was constructed based on the main objective of the decision-making process. Then, the fuzzy pairwise comparison matrices for the four risk factors O, S, D, and C for the objective were established by the three decision-makers  $DM_1$ ,  $DM_2$ , and  $DM_3$  considering Table 2. Afterward, for validating each judgment matrix, the consistency of each fuzzy pairwise comparison matrix constructed in the previous step was checked by employing the method proposed by Gogus and Boucher (1998). In the mentioned case study, all of these fuzzy pairwise comparison matrices were acceptable because the values of  $CR^m$  and  $CR^g$  obtained for them were less than 0.1. Finally, the weights of the four risk factors were computed by implementing Steps 4, 5, 6, and 7 of the proposed extended fuzzy AHP method, in turn. Table 6 demonstrates the fuzzy pairwise comparison matrices established by the three DMs and the weights of the four risk factors. As can be observed from Table 6, the severity (S) and occurrence (O) respectively have higher weight and importance than the other risk factors in this case study.

After calculating the weights of the four risk factors, the fuzzy weighted aggregated values of the risk factors were computed for each toxic gas by multiplying the fuzzy aggregated value of each risk factor by the weight of the risk factor utilizing Eq. (5). Subsequently, the fuzzy weighted aggregated risk priority number (FWARNP) was calculated for each toxic gas by multiplying the fuzzy weighted aggregated values of the four risk factors using Eq. (4). In the final step, the identified toxic gases were prioritized by comparing the achieved FWARNPs

employing the proposed novel fuzzy numbers ranking method. Table 7 illustrates the FWARPN and rank of each toxic gas. According to Table 7, in this plant, the carbonyl chloride (phosgene) ( $COCl_2$ ), carbon monoxide ( $CO$ ), and chlorine ( $Cl$ ) respectively have higher FWARPN, risk, and priority than the other detected toxic gases, thus, suitable actions should be implemented to deal with them sooner than the others. Furthermore, this table shows the fuzzy aggregated risk priority number (FARPN) and rank of each toxic gas calculated by the traditional fuzzy FMEA method. As can be observed from Table 7, the FWARPNs and ranks obtained by the proposed fuzzy FMEA model are different from the FARPNs and ranks achieved by the traditional fuzzy FMEA method for toxic gases. Therefore, these differences indicate the importance of using the weights of the risk factors in the suggested fuzzy FMEA model.

**Table 5.** Fuzzy values of the four risk factors are determined by the three DMs for each toxic gas.

| Toxic gas | DM     | Occurrence (O) | Severity (S) | Detection (D) | Cost (C)  |
|-----------|--------|----------------|--------------|---------------|-----------|
| $SO_2$    | $DM_1$ | (1,1,2)        | (6,7,8)      | (6,7,8)       | (4,5,6)   |
|           | $DM_2$ | (1,1,2)        | (5,6,7)      | (5,6,7)       | (5,6,7)   |
|           | $DM_3$ | (1,1,2)        | (5,6,7)      | (5,6,7)       | (6,7,8)   |
| $C_2N_2$  | $DM_1$ | (1,2,3)        | (5,6,7)      | (7,8,9)       | (4,5,6)   |
|           | $DM_2$ | (2,3,4)        | (5,6,7)      | (8,9,10)      | (5,6,7)   |
|           | $DM_3$ | (1,2,3)        | (5,6,7)      | (8,9,10)      | (4,5,6)   |
| $Cl$      | $DM_1$ | (1,1,2)        | (8,9,10)     | (6,7,8)       | (7,8,9)   |
|           | $DM_2$ | (1,1,2)        | (8,9,10)     | (7,8,9)       | (8,9,10)  |
|           | $DM_3$ | (1,1,2)        | (8,9,10)     | (7,8,9)       | (7,8,9)   |
| $NO_2$    | $DM_1$ | (1,1,2)        | (6,7,8)      | (8,9,10)      | (6,7,8)   |
|           | $DM_2$ | (1,1,2)        | (5,6,7)      | (8,9,10)      | (6,7,8)   |
|           | $DM_3$ | (1,1,2)        | (6,7,8)      | (9,10,10)     | (7,8,9)   |
| $COCl_2$  | $DM_1$ | (1,2,3)        | (9,10,10)    | (8,9,10)      | (9,10,10) |
|           | $DM_2$ | (1,1,2)        | (9,10,10)    | (9,10,10)     | (8,9,10)  |
|           | $DM_3$ | (1,2,3)        | (9,10,10)    | (9,10,10)     | (9,10,10) |
| $H_2S$    | $DM_1$ | (1,2,3)        | (5,6,7)      | (7,8,9)       | (5,6,7)   |
|           | $DM_2$ | (1,2,3)        | (5,6,7)      | (7,8,9)       | (4,5,6)   |
|           | $DM_3$ | (1,2,3)        | (5,6,7)      | (7,8,9)       | (6,7,8)   |
| $CO$      | $DM_1$ | (1,1,2)        | (9,10,10)    | (5,6,7)       | (8,9,10)  |
|           | $DM_2$ | (1,2,3)        | (8,9,10)     | (5,6,7)       | (7,8,9)   |
|           | $DM_3$ | (1,1,2)        | (8,9,10)     | (6,7,8)       | (9,10,10) |

**Table 6.** Fuzzy pairwise comparison matrices for the four risk factors and the weight of each risk factor.

| Criterion      | DM     | Occurrence (O) | Severity (S)  | Detection (D) | Cost (C)    | Weight |
|----------------|--------|----------------|---------------|---------------|-------------|--------|
| Occurrence (O) | $DM_1$ | (1,1,1)        | (2/3,1,1)     | (1,1.5,2)     | (1,1,1.5)   | 0.250  |
|                | $DM_2$ | (1,1,1)        | (1/2,2/3,1)   | (1,1.5,2)     | (1,1,1.5)   |        |
|                | $DM_3$ | (1,1,1)        | (1/2,2/3,1)   | (1,1.5,2)     | (1,1,1)     |        |
| Severity (S)   | $DM_1$ | (1,1,1.5)      | (1,1,1)       | (1.5,2,2.5)   | (1,1.5,2)   | 0.359  |
|                | $DM_2$ | (1,1.5,2)      | (1,1,1)       | (2,2.5,3)     | (1.5,2,2.5) |        |
|                | $DM_3$ | (1,1.5,2)      | (1,1,1)       | (2,2.5,3)     | (1,1.5,2)   |        |
| Detection (D)  | $DM_1$ | (1/2,2/3,1)    | (2/5,1/2,2/3) | (1,1,1)       | (2/3,1,1)   | 0.170  |
|                | $DM_2$ | (1/2,2/3,1)    | (1/3,2/5,1/2) | (1,1,1)       | (2/3,1,1)   |        |
|                | $DM_3$ | (1/2,2/3,1)    | (1/3,2/5,1/2) | (1,1,1)       | (1/2,2/3,1) |        |
| Cost (C)       | $DM_1$ | (2/3,1,1)      | (1/2,2/3,1)   | (1,1,1.5)     | (1,1,1)     | 0.221  |
|                | $DM_2$ | (2/3,1,1)      | (2/5,1/2,2/3) | (1,1,1.5)     | (1,1,1)     |        |
|                | $DM_3$ | (1,1,1)        | (1/2,2/3,1)   | (1,1.5,2)     | (1,1,1)     |        |

After calculating the weights of the four risk factors, the fuzzy weighted aggregated values of the risk factors were computed for each toxic gas by multiplying the fuzzy aggregated value of each risk factor by the weight of the risk factor utilizing Eq. (5). Subsequently, the fuzzy weighted aggregated risk priority number (FWARNP) was calculated for each toxic gas by multiplying the fuzzy weighted aggregated values of the four risk factors using Eq. (4). In the final step, the identified toxic gases were prioritized by comparing the achieved FWARNPs employing the proposed novel fuzzy numbers ranking method. Table 7 illustrates the FWARNP and rank of each toxic gas. According to Table 7, in this plant, the carbonyl chloride (phosgene) ( $COCl_2$ ), carbon monoxide ( $CO$ ), and chlorine ( $Cl$ ) respectively have higher FWARNP, risk, and priority than the other detected toxic gases, thus, suitable actions should be implemented to deal with them sooner than the others. Furthermore, this table shows the fuzzy aggregated risk priority number (FARNP) and rank of each toxic gas calculated by the traditional fuzzy FMEA method. As can be observed from Table 7, the FWARNPs and ranks obtained by the proposed fuzzy FMEA model are different from the FARNPs and ranks achieved by the traditional fuzzy FMEA method for toxic gases. Therefore, these differences indicate the importance of using the weights of the risk factors in the suggested fuzzy FMEA model.

**Table 7.** Fuzzy weighted aggregated risk priority number (FWARNP), fuzzy aggregated risk priority number (FARNP) and ranks of each toxic gas.

| Toxic gas | Fuzzy weighted aggregated risk priority number (FWARNP) | Rank | Fuzzy aggregated risk priority number (FARNP) | Rank |
|-----------|---------------------------------------------------------|------|-----------------------------------------------|------|
| $SO_2$    | (0.479560,0.811506,2.538672)                            | 7    | (142.2222,240.6667,752.8889)                  | 7    |
| $C_2N_2$  | (0.746815,2.181999,4.816832)                            | 4    | (391.1111,575.0000,1617.778)                  | 3    |
| $Cl$      | (1.318790,1.938847,5.454997)                            | 3    | (221.4815,647.1111,1428.518)                  | 4    |
| $NO_2$    | (1.008450,1.538589,4.308548)                            | 6    | (175.0000,576.0000,1323.000)                  | 5    |
| $COCl_2$  | (2.279409,5.251434,8.991753)                            | 1    | (676.0000,1557.407,2666.667)                  | 1    |
| $H_2S$    | (0.590084,1.942219,4.461034)                            | 5    | (299.0741,456.2963,1277.778)                  | 6    |
| $CO$      | (1.198900,2.391806,5.577385)                            | 2    | (355.5556,709.3333,1654.074)                  | 2    |

## Conclusion

Risk assessment is a fundamental step for controlling different risks and the effects of them in a wide range of industries and organizations. One of the most well-known risk assessment tools is the FMEA method. Despite the widespread application of the conventional FMEA method, it has major deficiencies. In this research, the suggested novel fuzzy AHP-based FMEA model was introduced for overcoming the drawbacks of the classical FMEA and analyzing the risks arising from different failure modes more precisely. To achieve these goals, in the proposed hybrid fuzzy model, the fuzzy weighted aggregated risk priority number (FWARNP), as a new risk assessment criterion, was applied instead of the RPN for assessing the risk of each failure mode. Also, the parameter of cost (C) (the required cost for eliminating the effects of failure occurred), as a new economic parameter, was added to the three previous safety parameters of occurrence (O), severity (S), and detection (D). Therefore, the FWARNPs were calculated by employing the four risk parameters O, S, D, and C. Also, the proposed extended fuzzy AHP method was utilized for computing the weights of these four risk parameters. Moreover, the suggested novel fuzzy numbers ranking method was used in both the proposed fuzzy FMEA and AHP methods for enhancing their efficiency. The presented comparative example confirmed the validity of this fuzzy numbers ranking method. Eventually, to show the practicability and effectiveness of the suggested model, Kerman Ghete Gostar Casting Plant was taken into consideration as a case study, in which the risks of toxic gases release were evaluated by the proposed fuzzy FMEA model. Findings demonstrated that in this plant, the

carbonyl chloride (phosgene) ( $COCl_2$ ), carbon monoxide ( $CO$ ), and chlorine ( $Cl$ ) respectively had higher FWARPN, risk, and priority than the other identified toxic gases, therefore, appropriate actions should be performed to deal with them sooner than the others. In the end, the results provided by the proposed fuzzy AHP-based FMEA model were compared with the results of the conventional fuzzy FMEA method. This comparison showed that the FWARPNs and the ranks achieved by the suggested fuzzy FMEA model were different from the FARPNS and the ranks obtained by the conventional fuzzy FMEA method for the detected toxic gases. These differences were due to the use of the weights of the risk factors in the proposed fuzzy AHP-based FMEA model. Thus, the importance of utilizing these weights was indicated.

For further studies, the proposed hybrid fuzzy model can be employed for new case studies in different industries in various countries. Furthermore, the use of new MCDM methods in the suggested hybrid model can be taken into consideration to obtain the weights of the risk factors in future studies. Also, another new fuzzy numbers ranking method can be applied in both the proposed fuzzy FMEA and AHP methods to compare the fuzzy numbers. Moreover, in future studies, other important risk factors related to other aspects can be added to the four risk factors of occurrence (O), severity (S), detection (D), and cost (C) considered in this study.

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