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*Research Article*

# Characterizing Solution for Stock Portfolio Problem via Pythagorean Fuzzy Approach

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**Abstract**
In this paper, a stock portfolio problem is formulated as a linear programming problem. The problem is considered by incorporating pythagorean fuzzy numbers in all of rate of risked return, expected return rate, portfolio risk amount, and interest rate of the bank. After converting the problem into the corresponding crisp based on the score function, a solution procedure is suggested to give the decision of the portfolio investment combined with investors in savings and securities. The advantages of this study are the investor is freely to choose the risk coefficients enable him/ her to maximize the expected returns; also, the investor may determine his/ her strategies under consideration of his/ her own conditions. GAMS software used for obtaining the optimal return rate. An example is introduce to clarify the practically and the efficiency of the technique.

**Keywords**: Stock portfolio, Pythagorean fuzzy numbers, Score function, Pythagorean fuzzy optimal return rate

## Introduction

The portfolio optimization is one of the fundamental problems in asset management that aims to reduce the risk of an investment by diversifying it into assets expected to fluctuate independently (Elton et al., 2009). A portfolio is a grouping of financial assets such as stocks, bonds, commodities, currencies and cash equivalents, as well as their funds counterparts, including mutual, exchange- traded and closed funds. A portfolio can also consist of non-publicly tradable securities, like real estate, art, and private investment. Portfolio are held directly by investors and/ or managed by financial professionals and money managers (Azizah, 2017). Skrinjaric and Sego (2018) applied Grey Relational Analysis (GRA) approach to evaluate the performance on a sample of stocks by considering many different factors such as the market factors, return distribution characteristics and financial statements information. Fakher and Abedi (2017) studied the effect of the environmental quality, the index foreign investment, and trade openness on economic growth in selected developing countries.

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Fuzzy Set Theory introduced by Zadeh (1965) has been widely used to solve many practical problems, including financial risk management, since it allows us to describe and treat imprecise and uncertain elements present in a decision problem. Then the imperfect knowledge of the returns on the assets and the uncertainty involved in the behavior of financial markets may also be introduced by means of fuzzy quantities and/ or fuzzy constraints. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers. Dubois and Prade (1980) extended the use of algebraic operations on real numbers to fuzzy numbers by use of a fuzzification principle

Portfolio selection (PS) problem is equivalent to the investor selecting the optimal portfolio from a set of possible portfolios Also, it focus on the optimal allocation of one's wealth to obtain maximum profitable return under minimum risk control (Gao and Liu, 2017). Due to the uncertainty of the real- world applications, the actual return of each security cannot be prespecified in advance. The theory of optimal portfolios has been developed by Markowitz (1952), where he has firstly proposed the mean-variance models in which the variance was used to quantify the existing risks in the uncertain return. The PS problem is typically a linear programming model when the return of each security is constant. Wu and Li (2011) investigated a multi- period mean- variance portfolio selection with regime switching and uncertain exit time. Liu and Qin (2012) investigated a mean semi-absolute deviation model for uncertain PS. A portfolio-adjusting problem was proposed by Huang and Ying (2013). Goldfarb and Iyengar (2003) formulated and solved PS problems. Simamora and Sashanti (2013) proposed two fuzzy PS models: The first model having L-R fuzzy return rates and the second are introduced with return and risk characterized by interval- valued means. Sardou et al. (2015) solved multi- objective problem of PS based on multi- objective fuzzy approach and genetic algorithm. Kumar et al. (2014) introduced PS with budget limitation.

Makhatabrafiei and Fatahzadeh (2013) applied multi- objective model of linear regression to optimal PS with the objectives of risk minimization and capital return maximization. Ashrafzadeh et al. (2016) developed a new fuzzy multi- objective programming model for optimal PS selection under uncertainty based on mean-variance- skewness model. Banihashemi et al. (2018) solved PS problem using multi- objective model considering four parameters: Expected return, downside beta coefficient, semi variance, and conditional value at risk at a specified confidence level. Li and Ng (2000) derived an analytical optimal solution for the multi- period mean- variance model. Ammar and Khalifa (2003) formulated a fuzzy portfolio optimization problem a programming problem.

Wei and Ye (2007) take into your consideration bankruptcy constraints in stochastic markets for solving a multi-period mean- variance PS model. Khalifa and Zein Eldin (2014) studied PS problem with fuzzy objective function coefficients and applied fuzzy programming approach to obtain the $\alpha-$optimal compromise. Luengo (2010) studied the PS problem under fuzzy environment. Bermudez et al. (2012) formulated the PS problem as a tri- objective optimization problem to find tradeoffs between risk, return, and the number of securities in the portfolio. Xu and Li (2002) introduced a class of stochastic multi-objective problems with one quadratic, several linear objective functions and linear constraints and its application on the PS model. Chen et al. (2011) proposed a model for optimal portfolio selection with mean-variance- skewness in intuitionist fuzzy environment. Nassif et al. (2013) presented a method for selecting the projects in the Information Technology area with support decision- making based on fuzzy logic. Zhang et al. (2016) studied a multi- period fuzzy PS problem maximizing the terminal wealth imposed by risk control. Wang and Kim (2019) proposed a model for the portfolio management based on the least squares support vector machine- mean- variance and design an algorithm to illustrate the performance of the model using the historical data from Shanghai stock exchange. Ramli and Jaaman (2019) introduced seven types of extended mean- variance method of portfolio of portfolio selection model based on the possibilitc return and possibilistics risk. Tian et al. (2019) proposed new risk measurement methods to describe or measure the real investments risks. Wei et al. (2019) showed that the feasibility of replacing expert subjective with knowledge resulted from data and determine the risk based on mining information from system operation data.

Stock price assessment, selection of optimum combination, and measure the risk of a portfolio investment is one important issue of investors. Azizah et al. (2017) used a single index model for the assessment of stock formulation optimization model using Lagrange multipliers technique that is to determine the proportion of assets to be invested. Lindberg (2009) introduced new parameterization of the drift rates to modify the $n$ stock Black- Scholes model, and solved Markowitz' continuous time PS in this framework. Daryaei et al. (2019) examined the empirical effects of stock trade- total value and other factors on nitrogen dioxide emission in Iran.

The rest of the paper is organized as follows: Section2 introduces the basic concepts and results related to fuzzy numbers, and Pythagorean fuzzy numbers. Section3 presents assumptions and notation needed in the paper. Section4 formulate stock portfolio optimization as Pythagorean fuzzy linear programming problem. Section 5 proposes solution procedure for solving the problem. Section 6 provides a numerical example to illustrate the efficiency of the solution technique. Finally, some concluding remarks are reported in section 7.

## Preliminaries

In this section, basic concepts and results related to fuzzy numbers, and octagonal fuzzy numbers and some of arithmetic operations are recalled

*Definition 1.* (Yager, 2014; Refomat and Yager, 2014). Let X be a fixed set, a Pythagorean fuzzy set is as defined as

$$\mathrm{P} = \{\langle \mathrm{x}, (\alpha_{\mathrm{P}}(\mathrm{x}), \beta_{\mathrm{P}}(\mathrm{x}))\rangle : \mathrm{x} \in \mathrm{X}\}.$$

Where, $_{\mathrm{P}}(\mathrm{x}): \mathrm{X} \rightarrow [0,1]$, and $\beta_{\mathrm{P}}(\mathrm{x}): \mathrm{X} \rightarrow [0,1]$are the degree of membership and non- membership functions, respectively. In addition, it holds that:

$$\left(_{\mathrm{P}}(\mathrm{x})\right)^2 + \left(\beta_{\mathrm{P}}(\mathrm{x})\right)^2 \leq 1.$$

*Definition 2.* (Yager, 2014; Refomat and Yager, 2014). Let $\tilde{\mathrm{a}}^{\mathrm{P}} = \left(\alpha_{\mathrm{i}}^{\mathrm{P}},\ \beta_{\mathrm{k}}^{\mathrm{P}}\right)$ and $\tilde{\mathrm{b}}^{\mathrm{P}} = \left(\alpha_{\mathrm{l}}^{\mathrm{P}},\ \beta_{\mathrm{s}}^{\mathrm{P}}\right)$ be two Pythagorean fuzzy numbers (PFN). Then, the arithmetic's operations are as follows

(i) $\tilde{\mathrm{a}}^{\mathrm{P}} \oplus \tilde{\mathrm{b}}^{\mathrm{P}} = \left(\sqrt{(\alpha_{\mathrm{i}}^{\mathrm{P}})^2 + (\alpha_{\mathrm{l}}^{\mathrm{P}})^2 - (\alpha_{\mathrm{i}}^{\mathrm{P}})^2.(\alpha_{\mathrm{l}}^{\mathrm{P}})^2}, \beta_{\mathrm{k}}^{\mathrm{P}}.\beta_{\mathrm{s}}^{\mathrm{P}}\right),$

(ii) $\tilde{\mathrm{a}}^{\mathrm{P}} \otimes \tilde{\mathrm{b}}^{\mathrm{P}} = \left(\beta_{\mathrm{i}}^{\mathrm{P}}.\beta_{\mathrm{l}}^{\mathrm{P}},\ \sqrt{(\alpha_{\mathrm{k}}^{\mathrm{P}})^2 + (\alpha_{\mathrm{s}}^{\mathrm{P}})^2 - (\alpha_{\mathrm{k}}^{\mathrm{P}})^2.(\alpha_{\mathrm{s}}^{\mathrm{P}})^2}\right),$

(iii) $\mathrm{k} \odot \tilde{\mathrm{a}}^{\mathrm{P}} = \left(\sqrt{1 - (1 - \alpha_{\mathrm{i}}^{\mathrm{P}})^{\mathrm{k}}}, \left(\beta_{\mathrm{k}}^{\mathrm{P}}\right)^{\mathrm{k}}\right), \mathrm{k} \succ 0.$

*Definition 3.* (Yager, 2014; Refomat and Yager, 2014). Let $\tilde{a}^P = (\alpha_i^P, \beta_k^P)$ and $\tilde{b}^P = (\alpha_l^P, \beta_s^P)$ be two PFNs. Then

(i) Score function: $S(\tilde{a}^P) = \frac{1}{2}\left(1 - (\alpha_i^P)^2 - (\beta_k^P)^2\right)$.

(ii) Accuracy function: $A(\alpha_i^P) = (\alpha_i^P)^2 + (\beta_k^P)^2$.

*Definition 4.* (Yager, 2014; Refomat and Yager, 2014). Let$\tilde{a}^P$, and $\tilde{b}^P$ be any two PFN, then

(i) $\tilde{a}^P(\succ)\tilde{b}^P$ if and only if $S(\tilde{a}^P)(>)S(\tilde{b}^P)$,

(ii) $\tilde{a}^P(\prec)\tilde{b}^P$ if and only if $S(\tilde{a}^P) < S(\tilde{b}^P)$,

(iii) $S(\tilde{a}^P) = S(\tilde{b}^P)$, and $A(\tilde{a}^P) < A(\tilde{b}^P)$ then $\tilde{a}^P(\prec)\tilde{b}^P$,

(iv) $S(\tilde{a}^P) = S(\tilde{b}^P)$, and $A(\tilde{a}^P) > A(\tilde{b}^P)$ then $\tilde{a}^P(\succ)\tilde{b}^P$,

(v) $S(\tilde{a}^P) = S(\tilde{b}^P)$, and $A(\tilde{a}^P) = A(\tilde{b}^P)$ then $\tilde{a}^P(\approx)\tilde{b}^P$.

## Assumptions and Notation

*Assumptions*

In the stock portfolio problem, the following assumptions are used

- The expected rate of return and risk loss rate are evaluated by investors;
- Indefinite and divided Securities;
- No need to pay the transaction costs in the course of transaction;
- Assumptions of non- satisfaction and avoiding risk are obeyed by investors;
- Allowable short selling operation ;
- Interest rate of the bank is unchanged for the investors during the investment period.

*Notation*

In the stock problem, the following notation can be used

***Index:***

j: Risk securities

***Decision variable:***

$x_j$: The funds proportion of the secondary securities;

$x_0$: The total amount proportion in the investment period;

x: Return rate expectation of the combined investment

***Parameters:***

A: Rate of risked return

$r_j$: Expected return rate

B: Portfolio risk amount

$r_0$: Interest rate of the bank

## Problem statement and solution concepts

In this section, we introduced the stock portfolio investment problem introduced by (Yin, 2018) in fuzzy environment as

$$\text{maximize } \tilde{R}^P = \tilde{r}_0^P x_0 + \sum_{j=1}^{n} \tilde{r}_j^P x_j$$

Subject to (1)

$$\widetilde{A}^P x \le \widetilde{B}^P,$$
$$x_0 + \sum_{j=1}^{n} x_j = 1,$$
$$x_j \ge 0, j = 1,2, \dots, n.$$

Where, $\widetilde{A}^P = \left(\tilde{a}^P_{ij}\right)_{m\times n}, \widetilde{B}^P = \left(\tilde{b}^P_1, \dots, \tilde{b}^P_m\right)^T, \tilde{r}^P = (\tilde{r}^P_1, \ \dots, \tilde{r}^P_n)^T, x = (x_1, \dots, x_n)^T,$

Let, $\widetilde{X} = \left\{x: \widetilde{A}^P x \le \widetilde{B}^P, x_0 + \sum_{i=1}^{n} x_i = 1, x_j \ge 0, j = \overline{1, n}\right\}.$

*Definition 5.* The point$x$, which satisfies the condition in model (1), called Pythagorean fuzzy optimization solution of model (1).

Let us denote that: $\tilde{a}^P_{ij} = \left(\tilde{\alpha}^P_{ij}, \tilde{\delta}^P_{ij}\ \right), \tilde{b}_i = \left(\tilde{\beta}^P_i, \tilde{\gamma}^P_i\right), \tilde{r}_i = \left(\tilde{\epsilon}^P_i, \tilde{\vartheta}^P_i\right)$. Then, Model1 becomes

$$\text{maximize } \widetilde{R}^P = \left(\tilde{\epsilon}^P_0, \tilde{\vartheta}^P_0\right)x_0 + \sum_{j=1}^{n}\left(\tilde{\epsilon}^P_i, \tilde{\vartheta}^P_i\right) x_j$$

Subject to (2)

$$\sum_{j=1}^{n}\left(\tilde{\alpha}^P_{ij}, \tilde{\delta}^P_{ij}\ \right) x_j \le \left(\tilde{\beta}^P_i, \tilde{\gamma}^P_i\right), i = 1, 2, \dots, m,$$
$$x_0 + \sum_{j=1}^{n} x_j = 1,$$
$$x_j \ge 0, j = \overline{1, n}.$$

Based on the definition3, model (2) becomes

$$\text{maximize } R = r_0 x_0 + \sum_{j=1}^{n} r_j\, x_j$$

Subject to (3)

$$Ax \le B,$$
$$x_0 + \sum_{j=1}^{n} x_j = 1,$$
$$x_j \ge 0, j = j = \overline{1, n}\ .$$

**Solution procedure**

The steps of the solution procedure that premises the fuzzy optimal solution for the model (1) are as follow

Step1: Formulate the model (1),

Step2: Represent the model (1) as in model (2),

Step3: Convert the model (2) into the corresponding crisp form model (3) based the definition3,

Step4: Using the GAMS software for obtaining the pythagorean fuzzy optimal return,

Step5: Stop.

**Numerical example**

Consider four stocks with the following data:

Expected return rate%:
$\tilde{r}_1^P = (0.9, 0.1)$, $\tilde{r}_2^P = (0.8, 0.3)$, $\tilde{r}_3^P = (0.6, 0.3)$, $\tilde{r}_4^P = (0.7, 0.2)$.
Risk loss rate %:
$\tilde{a}_{11}^P = (0.9, 0.2), \tilde{a}_{12}^P = (0.4, 0.8), \quad \tilde{a}_{13}^P = (0.6, 0.5), \tilde{a}_{14}^P = (0.8, 0.4),$
$\tilde{a}_{21}^P = (0.7, 0.4), \tilde{a}_{22}^P = (0.8, 0.3), \tilde{a}_{23}^P = (0.4, 0.6)\ \tilde{a}_{24}^P = (0.5, 0.6),$
$\tilde{a}_{31}^P = (0.9, 0.3), \tilde{a}_{32}^P = (0.4, 0.7), \tilde{a}_{33}^P = (0.8, 0.4)\ \tilde{a}_{34}^P = (0.5, 0.6),$
$\tilde{a}_{41}^P = (0.3, 0.9), \tilde{a}_{42}^P = (0.9, 0.4)\ \tilde{a}_{43}^P = (0.8, 0.5), \tilde{a}_{44}^P = (0.4, 0.9).$
Risk coefficient %:
$\tilde{b}_1^P = (0.4, 0.9), \tilde{b}_2^P = (0.9, 0.1), \tilde{b}_3^P = (0.9, .02), \tilde{b}_4^P = (0.8, 0.5).$
For $\tilde{r}_0^P = (0.4, 0.8)$.

*Step1:* Formulate the problem according to model (1) as

$$\begin{aligned}
&\text{maximize } \widetilde{\mathrm{R}}^{\mathrm{P}} = \tilde{r}_0^P \otimes \mathrm{x}_0 \oplus \sum_{\mathrm{j}=1}^{4} \tilde{\mathrm{r}}_{\mathrm{j}}^{\mathrm{P}} \otimes \mathrm{x}_{\mathrm{j}} \\
&\text{Subject to} \\
&\widetilde{\mathrm{A}}^{\mathrm{P}}\mathrm{x} \leq \widetilde{\mathrm{B}}^{\mathrm{P}}, \\
&\mathrm{x}_0 + \sum_{\mathrm{j}=1}^{4} \mathrm{x}_{\mathrm{j}} = 1; \quad \mathrm{x}_{\mathrm{j}} \geq 0, \mathrm{j} = 0,1,2,3,4.
\end{aligned} \tag{4}$$

*Step2*: Rewrite the problem (2) as

$$\begin{aligned}
&\text{maximize } \widetilde{\mathrm{R}}^{\mathrm{P}} = \left(\tilde{\epsilon}_0^{\mathrm{P}}, \tilde{\vartheta}_0^{\mathrm{P}}\right) \otimes \mathrm{x}_0 \oplus \sum_{\mathrm{j}=1}^{4} \left(\tilde{\epsilon}_{\mathrm{i}}^{\mathrm{P}}, \tilde{\vartheta}_{\mathrm{i}}^{\mathrm{P}}\right) \otimes \mathrm{x}_{\mathrm{j}} \\
&\text{Subject to} \\
&\sum_{\mathrm{j}=1}^{5} \left(\tilde{\alpha}_{\mathrm{ij}}^{\mathrm{P}}, \tilde{\delta}_{\mathrm{ij}}^{\mathrm{P}}\ \right) \mathrm{x}_{\mathrm{j}} (\preccurlyeq) \left(\tilde{\beta}_{\mathrm{i}}^{\mathrm{P}}, \tilde{\gamma}_{\mathrm{i}}^{\mathrm{P}}\right), \mathrm{i} = 1, 2, 3, 4, \\
&\mathrm{x}_0 + \textstyle\sum_{\mathrm{j}=1}^{4} \mathrm{x}_{\mathrm{j}} = 1; \quad \mathrm{x}_{\mathrm{j}} \geq 0, \mathrm{j} = 0, 1, 2, 3, 4.
\end{aligned} \tag{5}$$

*Step3:* Convert problem (5) into the corresponding crisp problem

$$\begin{aligned}
&\text{maximize } R = 0.1x_0 + 0.09x_1 + 0.175\mathrm{x}_2 + 0.275\mathrm{x}_3 + 0.235\mathrm{x}_4 \\
&\text{Subject to} \\
&0.075x_1 + 0.10\mathrm{x}_2 + 0.195\mathrm{x}_3 + 0.10\mathrm{x}_4 \leq 0.015, \\
&0.175x_1 + 0.135\mathrm{x}_2 + 0.24\mathrm{x}_3 + 0.195\mathrm{x}_4 \leq 0.09, \\
&0.05x_1 + 0.175\mathrm{x}_2 + 0.1\mathrm{x}_3 + 0.19\mathrm{x}_4 \leq 0.075, \\
&0.04x_1 + 0.015\mathrm{x}_2 + 0.055\mathrm{x}_3 + 0.015\mathrm{x}_4 \leq 0.055, \\
&x_0 + x_1 + \mathrm{x}_2 + \mathrm{x}_3 + \mathrm{x}_4 = 1;\ x_j \geq 0, j = 0, 1,2,3,4.
\end{aligned} \tag{6}$$

*Step4*: Using the GAMS software to solve problem (6). The solution illustrated in the following table

**Table1**. The Pythagorean fuzzy optimal solution

| Optimal solution | Optimum value |
|---|---|
| $x_0 = 0.80$ | |
| $x_1 = 0.2$ | $R = 0.098$ |
| $x_2 = 0$ | $\widetilde{\mathrm{R}}^{\mathrm{P}} = (0.67768, 0.64979)$ |
| $x_3 = 0$ | |
| $x_4 = 0$ | |

## Concluding Remarks

In this paper, we discussed stock portfolio investment problem as a linear programming problem with Pythagorean fuzzy numbers in the entire rate of risked return, expected return rate, portfolio risk amount, and interest rate of the bank. The study under uncertainty makes the investment portfolio more realistic and practice to describe the expected return rate, and risk loss rate. The advantages of the method for the investors are the ability for choosing the risk coefficient to achieve higher expected returns, and determining his/ her strategies for selecting the portfolios.

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